OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4758/01

MATHEMATICS (MEI)

Differential Equations

QUESTION PAPER

FRIDAY 15 JUNE 2018: Afternoon

DURATION: 1 hour 30 minutes

plus your additional time allowance

MODIFIED ENLARGED 24pt

Candidates answer on the Printed Answer Book sent with the standard paper or any suitable paper supplied by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4758/01 sent with standard paper
MEI Examination Formulae and Tables (MF2) sent with standard paper
Insert for question 3(b)(ii) and 3(b)(iii)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (CE) 169487/1

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer any THREE questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 In this question, you may assume that $t^k e^{-t} \to 0$ as $t \to \infty$ for any constant k.

The differential equation $4\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 9x = f(t)$ is to be solved for $t \geqslant 0$.

Firstly consider the case $f(t) = 9t^2 - 3t - 1$.

(i) Find the general solution for x in terms of t. [9]

You are given that $x = 5$ and $\frac{dx}{dt} = 0$ when $t = 0$.

(ii) Find the particular solution. [4]

(iii) Show that x is positive for all values of $t \geqslant 0$. [3]

Now consider the case $f(t) = -48 \sin 2t - 14 \cos 2t$.

(iv) Find the general solution for x in terms of t. [6]

(v) Describe the behaviour of x for large values of t. [2]

2 **Take g as 10 in this question.**

A particle P of mass 0.1 kg is in a liquid and is projected vertically downwards. At time t s, the velocity of P is $v\,\mathrm{m\,s}^{-1}$ and the depth of P below its point of projection, O, is x m. The only forces on P are its weight and a resistance force R N. A scientist investigates two different models for R.

In the first model, the resistance is given by $R = 0.2v$ and the initial speed of P is $2\,\mathrm{m\,s}^{-1}$.

(i) Use this information to form a differential equation involving v and t. Solve the differential equation to show that $v = 5 - 3\mathrm{e}^{-2t}$. [7]

(ii) Sketch the graph of v against t. [2]

(iii) Find an expression for x in terms of t and hence find the depth of P below O when its speed is three-quarters of its terminal speed. [7]

In the second model, the resistance is given by $R = 0.0625v^2$ and the initial speed of P is again $2\,\mathrm{m\,s}^{-1}$.

(iv) Find v in terms of x. [6]

(v) State the terminal speed of P and find the depth of P below O when its speed is three-quarters of its terminal speed. [2]

3 (a) A curve in the x-y plane satisfies the differential equation

$$\frac{dy}{dx} - \frac{2y}{x} = x^k \sin 2x,$$

where k is a constant and $x > 0$.

Firstly consider the case $k = 3$.

(i) Find the general solution for y in terms of x. [7]

(ii) Given that $y = 0$ when $x = \frac{1}{4}\pi$, find the exact value of y when $x = \frac{1}{2}\pi$. [4]

Now consider the case $k = 2.5$.

(iii) Use Euler's method, with a step length of 0.1 and initial conditions $y = 0$ when $x = 0.5$, to estimate y when $x = 0.8$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$. [5]

(b) Solutions of the differential equation $\frac{dy}{dx} = x^2 - y$ are to be investigated using a tangent field.

(i) Show that the isocline for which $\frac{dy}{dx} = 1$ is a parabola. State the coordinates of its turning point. [2]

(ii) In your Answer Book, sketch on the given axes the isoclines for the cases $\frac{dy}{dx} = m$ for $m = 0, \pm 1, \pm 2$. Use these isoclines to draw a tangent field. [3]

(iii) Sketch the solution curve through (0, 1) and the solution curve through (1, 0). [3]

4 The simultaneous differential equations

$$\frac{dx}{dt} = 7x + 2y + 13e^{4t},$$

$$\frac{dy}{dt} = -9x + y + e^{7t}$$

are to be solved.

(i) Eliminate x to obtain a second order differential equation for y in terms of t. Hence find the general solution for y. **[12]**

(ii) Given that $y = -3$ and $\frac{dy}{dt} = 60$ when $t = 0$, find the particular solution for y. **[4]**

(iii) Find the corresponding particular solution for x. **[2]**

(iv) Find the smallest positive value of t for which $y = 0$. **[4]**

(v) Show that $\frac{y}{x} \to 0$ as $t \to \infty$. **[2]**

END OF QUESTION PAPER